The dynamic IS-LM model

We present here a dynamic version of the standard IS-LM model. As in the traditional static version of the model, it is considered the existence of two markets: the goods and service markets and the money market. The macroeconomic equilibrium in this setup is represented by the existence of equilibrium in both markets simultaneously. Additional to the IS equation and the LM equation presented in the static version, the model includes two additional equations accounting for the dynamic adjustment in output and prices.

The model economy is represented by the following four equations:

\begin{align*}
m_t - p_t &= \psi y_t - \theta i_t \quad (1) \\
y^d_t &= \beta_0 - \beta_1 (i_t - \Delta p^e_t) \quad (2) \\
\Delta p_t &= \mu (y_t - y^n_t) \quad (3) \\
\Delta y_t &= v (y^d_t - y_t) \quad (4)
\end{align*}

where all variables are defined in logarithms, except the nominal interest rate, and where \( m \) denotes the quantity of money, \( p \) is the price level, \( y \) is output, \( i \) is the nominal interest rate, \( y^d \) is the aggregate demand, and \( y^n \) is potential output. \( \Delta p^e \) represents expected inflation. Since we solve the model in a context of perfect foresight and under the assumption that expectations are rational, then we have that \( \Delta p^e = \Delta p \). The symbol \( \Delta \) defines the variation of the corresponding variable between two periods, where inflation is defined as

\[
\Delta p_t = p_{t+1} - p_t
\]

and output growth as

\[
\Delta y_t = y_{t+1} - y_t
\]
Equation (1) is the equilibrium condition in the money market, where the real balances (left side), depend positively on the level of production (transaction money demand) and negatively on the nominal interest rate (speculative money demand). Equation (2) represents the aggregate demand of the economy, which depends positively on an autonomous component (which we assume reflects government spending), which we assume is an exogenous variable, and negatively of the real interest rate. The real interest rate is represented by the approximation to the Fisher equation and is obtained as the difference between the nominal interest rate and the expected inflation rate. The quantity of money is assumed to be exogenous.

Apart from the two equilibrium equations for the two markets, the model is also composed of two dynamic equations that indicate the dynamic behavior of two endogenous variables (price level and production level) over time. Equation (3) indicates how prices move over time based on the differences between the level of output and potential output, where potential output is exogenously given. This equation can be interpreted as a version of the Phillips curve. If the production level is greater than the potential, then this equation is positive, so that prices increase (positive inflation). On the contrary, if the production level is below the potential, the equation would have a negative sign, indicating that prices would decrease (deflation). Finally, equation (4) is similar but representing the dynamics of the production level. This expression indicates how changes in the level of output (the growth rate of the economy) moves depending on the differences between aggregate supply and demand. If the aggregate-demand level is higher than the production level, the expression would take a positive value, indicating that the level of production increases. On the contrary, if the production level is higher than the demand, then the expression would take a negative value, so that the production level of the economy would decrease.

All parameters (represented by greek letters) are defined in positive terms. The parameter $\psi$ represents the elasticity of the real balances with respect to the level of production. $\theta$ is the semi-elasticity of the demand for money with respect to the nominal interest rate. It is a semi-elasticity because all the variables of the model are defined in logarithmic terms, except the interest rate, which, as it is a percentage, logarithms cannot be applied to it, since it is as if it were in that term. The parameter $\beta_1$ represents the elasticity of the level of aggregate demand with respect to the real interest rate, while $\beta_0$ is the autonomous component of aggregate demand, which we assume reflects public spending (public consumption). The parameter $\mu$ is the speed
of adjustment of prices to differences between the level of production and the level of potential production. Finally, the parameter \( v \) indicates the speed of adjustment of the level of output in response to differences between the level of aggregate demand and the production level of the economy.

**Solution** The model can be represented by the following system of two difference equations:

\[
\begin{bmatrix}
\Delta p_t \\
\Delta y_t
\end{bmatrix} = \begin{bmatrix}
0 & \frac{\mu}{\theta} - \frac{\mu}{\theta} - 1 \\
-v \frac{\beta_1}{\theta} & \frac{\beta_1}{\theta} \psi - 1
\end{bmatrix} \begin{bmatrix} p_t \\ y_t \end{bmatrix} + \begin{bmatrix}
0 & 0 \\
v \frac{\beta_1}{\theta} & -v \frac{\beta_1}{\theta}
\end{bmatrix} \begin{bmatrix} m_t \\ y_t^n \end{bmatrix}
\] (7)

**Steady state** Steady state of the economy is defined as the value for the endogenous variables for which they remain constant over time, that is, inflation and output growth are zero. Steady state values, denoted by an upper bar on the variable, are given by:

\[
\bar{p} = \frac{\theta \beta_0}{\beta_1} + m - (\psi + \frac{\theta}{\beta_1})y^n
\] (8)

\[
\bar{y} = y^n
\] (9)

**Eigenvalues** The solution of the system shows global stability. Eigenvalues are,

\[
\lambda_1, \lambda_2 = \frac{v(\beta_1 \mu - \frac{\beta_1}{\theta} \psi - 1) \pm \sqrt{v(\beta_1 \mu - \frac{\beta_1}{\theta} \psi - 1)^2 - \frac{4v\beta_1}{\theta} \mu}}{2}
\] (10)