Investment-Specific Technological Change model

1 The model

Here we develop a Dynamic General Equilibrium model in which, additional to an aggregate productivity (TFP) shock, it is also specific technological progress associated to capital inputs, which depends on the investment process. This exercise differentiates neutral technological progress resulting from changes in TFP, and technological progress specific to capital and associated with the process of investing in new capital assets. Whereas the former is a change in the overall level of economic efficiency, the latter refers to the amount of technology that can be acquired with a production unit. In this exercise we will replicate, using the Excel Solver tool, the main result of the paper by Greenwood, Hercowitz and Krusell (2000), studying how the economy responds to investment-specific technological change shocks. Two changes are introduced with respect to the basic RBC model: First, the capital accumulation equation now accounts for changes in the quality of new vintages of capital through the investment process; second, a new shock must be defined for the investment-specific technical change (ISTC).

The feasibility restriction facing the economy is given by:

$$C_t + I_t = Y_t \tag{1}$$

where C_t is the consumption, I_t is the investment, and Y_t is the final output. The investment is accumulated in the form of physical capital, K_t, via the following process:

$$K_{t+1} = (1-\delta)K_t + Z_t I_t \tag{2}$$

where $\delta > 0$ is the rate of physical depreciation of capital, and Z_t represents technological progress specific to the investment. Z_t determines the amount of capital that can be purchased with a production unit, representing the current state of the technology to produce capital. In the standard neoclassical model we have to $Z_t = 1$ for all t, that is, the amount of capital that can be purchased with a final production unit is constant in time. However, in reality the relative price of capital falls broadly, evidence that over time we can buy a larger amount of capital with the same amount of final production. Thus, the higher Z_t, the greater the amount of capital that can be incorporated into the economy with an investment unit, reflecting how the quality of capital has increased.

To obtain a measure of technological progress specific to investment, it is necessary to have prices of capital assets adjusted for quality. This is what is called the hedonic price, i.e., the price of a particular capital asset whose quality remains constant over time. In order to make this comparison, we need quality-adjusted prices of the product. This phenomenon can be seen clearly in the case of computers. Suppose a computer today is about the same nominal price as a computer 20 years ago, but its power storage and computation is 1,000 times greater. It therefore appears that a computer today is 1,000 times cheaper, taking into account quality change, than a computer 20 years ago. Thus, the quality-adjusted price of fixed capital can be defined as:

$$\frac{1}{Z_t} \tag{3}$$

Households The household's maximization problem is given by:

$$\max_{\{C_t\}_{t=0}^{T}} E_t \sum_{t=0}^{T} \beta^t U(C_t) \tag{4}$$

where $\beta < 1$ is the intertemporal discount factor, and $E_t(\cdot)$ is the mathematical expectation of future variables. Given that we consider a context without uncertainty, that is, with perfect foresight, we can eliminate the expectation operator from the maximization problem, as the value of all the variables in the future is known at the present time.

The budget constraint is given by:

$$C_{t+1} + I_t = W_t L_t + R_t K_t \tag{5}$$

where W_t is the salary per unit of time, L_t is labor, and R_t is the return on capital. Taken into account the capital accumulation equation, the budget constraint is:

$$C_t + \frac{K_{t+1}}{Z_t} = W_t L_t + \frac{R_t + (1-\delta)}{Z_t} K_t \tag{6}$$

Finally, we need to determine the stock of initial capital, K_0, as well as the stock of final capital, K_{T+1}, as we assume that households are finite-lived.

Let us assume that the utility function has a logarithmic form. The Lagrangian auxiliary function is:

$$\mathcal{L} = \sum_{t=0}^{T} \left[\beta^t \ln(C_t) - \lambda_t \left(\frac{C_t + K_{t+1}}{Z_t} - W_t L_t - \frac{R_t + (1-\delta)}{Z_t} K_t \right) \right] \tag{7}$$

First order conditions, for $t = 0, 1, 2, ..., T$, are given by:

$$\frac{\partial \mathcal{L}}{\partial C_t} : \frac{\beta^t}{C_t} - \lambda_t = 0 \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} : \lambda_{t+1} \left[\frac{R_{t+1} + (1-\delta)}{Z_{t+1}} \right] - \lambda_t \frac{1}{Z_t} = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} : \frac{C_t + K_{t+1}}{Z_t} - W_t L_t - \frac{R_t + (1-\delta)}{Z_t} K_t = 0 \tag{10}$$

Solving for the Lagrange's parameter, we obtain the following optimal consumption path

$$C_{t+1} Z_{t+1} = \beta \left[Z_{t+1} R_{t+1} + 1 - \delta \right] Z_t C_t \tag{11}$$

The firms We assume that firms maximize profits, subject to the technological restriction. The aggregate production (technology) function has the following general form:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{12}$$

where Y_t is the level of aggregate production of the economy and A_t is total factor productivity (TFP). The parameter α represents the elasticity of output to capital. We assume the TFP is determined exogenously via the following process:

$$A_t = A_{t-1}^{\rho} \varepsilon_t^A \tag{13}$$

where $\rho < 1$ is an autoregressive parameter that measures the persistence of shocks that affect TFP, and ε_t^A is a disturbance term, which we can consider either stochastic and deterministic. Similarly, we assume that ISTC follows a similar process,

$$Z_t = Z_{t-1}^{\phi}\varepsilon_t^Z \tag{14}$$

where $\phi < 1$ is an autoregressive parameter that measures the persistence of ISTC shocks, denoted by ε_t^Z. In our case, we will consider it as an exogenous deterministic variable, whose value is one, except at the moment in which a technological shock occurs, taking a value different from one (higher than one for a shock that increases capital-embodied technical change, and lower than one for a shock decreasing capital-embodied technical change).

The problem solved by the firm consists in maximizing profits, such that:

$$\max \Pi_t = A_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - R_t K_t \tag{15}$$

First order conditions for profit maximization are:

$$\frac{\partial \Pi}{\partial K_t} = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t = 0 \tag{16}$$

$$\frac{\partial \Pi}{\partial L_t} = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} - W_t = 0 \tag{17}$$

resulting in:

$$R_t = \alpha A_t K_t^{\alpha} L_t^{1-\alpha} = \alpha \frac{Y_t}{K_t} \tag{18}$$

$$W_t = 1 - \alpha A_t K_t^{\alpha-1} L_t^{-\alpha} = (1-\alpha)\frac{Y_t}{L_t} \tag{19}$$

Equations of the model Competitive equilibrium consists of finding sequences of the $\{C_t, I_t, K_t, R_t, W_t, Y_t, A_t, Z_t\}_{t=0}^{T}$ variables such that the conditions that define equilibrium are satisfied. Labor is normalized to one ($L_t = 1$). In summary, the model of our economy would be composed of the following eight equations for eight unknowns:

$$Ct_{+1} Z_{t+1} = \beta \left[Z_{t+1} R_{t+1} + 1 - \delta \right] C_t Z_t \tag{20}$$

$$R_t = \alpha \frac{Y_t}{K_t} \tag{21}$$

$$W_t = (1-\alpha) Y_t = (1-\alpha) A_t K_t^{\alpha} \tag{22}$$

$$Y_t = A_t K_t^{\alpha} \tag{23}$$

$$K_{t+1} = (1-\delta)K_t + Z_t I_t \tag{24}$$

$$C_t + I_t = Y_t \tag{25}$$

$$A_t = A_{t-1}^{\rho} \varepsilon_t^{A} \tag{26}$$

$$Z_t = Z_{t-1}^{\varphi} \varepsilon_t^{Z} \tag{27}$$

Steady state Steady state of the economy is defined by:

$$\overline{R} = \frac{1-\beta+\beta\delta}{\beta} \tag{28}$$

$$\overline{K} = \left(\frac{(1-\beta+\beta\delta)}{\alpha\beta\overline{AZ}}\right)^{\frac{1}{\alpha-1}} \tag{29}$$

$$\overline{Y} = \overline{A}\left[\frac{\alpha\beta\overline{AZ}}{(1-\beta+\beta\delta)}\right]^{\frac{\alpha}{1-\alpha}} \tag{30}$$

$$\overline{I} = \frac{\delta}{\overline{Z}}\left(\frac{(1-\beta+\beta\delta)}{\alpha\beta\overline{AZ}}\right)^{\frac{1}{\alpha-1}} \tag{31}$$

$$\overline{C} = \overline{A}\left[\frac{\alpha\beta\overline{AZ}}{(1-\beta+\beta\delta)}\right]^{\frac{\alpha}{1-\alpha}} - \delta\left(\frac{(1-\beta+\beta\delta)}{\alpha\beta\overline{AZ}}\right)^{\frac{1}{\alpha-1}} \tag{32}$$

$$\overline{A} = 1 \tag{33}$$

$$\overline{Z} = 1 \tag{34}$$

This model can be solved using the "Solver" tool of Excel by first calibrating the parameters of the model, and introducing a guess solution for consumption. Hence, no analytical solution is needed and the initial equations of the model (utility function, the budget constraint and initial and final conditions) can directly be introduced in Excel.

2 Taking the model to Excel

The model is solved in the spreadsheet "ISTC.xlsx". Once the model has been solved analytically and the corresponding dynamic system has been obtained, then we will solve it computationally in Excel using the "Solver" tool. We start calibrating the value of the parameters of the model, which appear in cells "B4" to "B8". From these parameters and the steady-state expressions calculated above, we can obtain the steady-state values for the model variables, which appear in cells "B11" to "B17". If we place the cursor in cell "B11", the expression that appears is:

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=PTF*((1-Beta+Delta*Beta)/(Alpha*PTF*Z*Beta))^(Alpha/(Alpha-1))
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which is the one corresponding to the value of the steady-state production. Similarly, in cell "B14" we have introduced the expression corresponding to the steady-state value of the stock of capital, so the expression that appears in said cell is:

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=((1-Beta+Delta*Beta)/(Alpha*PTF*Z*Beta))^(1/(Alpha-1))
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Similarly, in cell "B15" we calculate the steady-state value of the consumption, in cell "B16" the steady-state value of the investment and in cell "B17" the steady-state value of the interest rate. Finally, in cell "B20", we assign the value of the total factor productivity technological change that we assume occurs in period 1, taking a zero value initially, and in cell "B21", the value of the ISTC shock.

The variables of the model are defined in the columns "F-M", where the values corresponding to the initial steady state appear in the period 0. The column "E" is the time index, column "F" is TFP, column "G" is ISTC, while the column "H" gives us the optimal path of consumption, which is the variable that we have to calculate. Column "I" is the investment, which is simply the difference between what is produced and what is consumed, column "J" is production, column "K" is the stock of capital, column "L" is the return on capital and finally the "M" column is the discounted utility. In cell "G4", the expression:

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=G3^Phi*EpsilonZ
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we have introduced with the objective of simulating a productivity shock in period 1. In cell "G5" the introduced expression is "`=G4^Phi`", since we assume that the shock takes a positive or negative value at time 1 and zero in the following periods. This expression is copied to the following cells in the column.

In cell "K3", the initial capital stock appears. For its part, in cell "K4" appears the expression:

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=(1-Delta)*K3+I3*G3
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where the stock of capital in each period of time is the stock of capital of the previous period, discounting the depreciation, plus the new capital that is incorporated, which is determined by savings. Finally, column "M" presents the value of the utility in discounted terms.

The sum of the discounted utilities is calculated in cell "M34", which will be the target cell to be maximized in the "Solver" tool. The solution to the problem is obtained by executing the "Solver", once we have defined the target cell to be maximized (the "M34"), the final condition ("`$K$34=K0`"), and the cells to change with the solution ("`$H$4:$H$33`"), similar to how it has been carried out in the previous chapters.

3 Exercises

1. Use the spreadsheet "ISCT.xlsx", The Model tab, and compute the effects of a simultaneous aggregate productivity shock and an ISTC shock. For instance, an increase of 1 percent. (Hint: change the values of cells "B20" and "B21" to 1.01).

2. Simulate the ISTC shock for different values of the parameters α and β. How the effects of this technological shock change depending on the value of these two parameters.