The Solow growth model

1 The model

One of the most popular macroeconomic models is the neoclassical Solow growth model. This is a non-micro-based model, so it does not include any optimality criteria, and its main assumption is that the saving rate of the economy is an exogenous variable. In this theoretical framework, the behavior of the economy over time is determined by the process of capital accumulation or neutral technological progress if we assume that the total factor productivity shows a growing trend over time. Specifically, the Solow growth model is reduced to a dynamic equation that indicates the evolution of the capital stock per capita over time. Given the assumption of exogenous saving rate, once we have the solution for the capital stock of the economy, we can in turn determine the rest of macroeconomic variables, given a technological restriction.

The model economy is represented by the following six equations:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

$$K_{t+1} = (1-\delta)K_t + I_t \tag{2}$$

$$I_t = sY_t \tag{3}$$

$$Y_t = C_t + I_t \tag{4}$$

$$L_t = L_{t-1}(1+n)^t \tag{5}$$

$$A_t = A_{t-1}\varepsilon_t^A \tag{6}$$

where Y is output, K is the capital stock, L is labor (population), I is investment, and A is total factor productivity (TFP). The parameter α is the output elasticity to capital, δ is the capital depreciation rate, s is the saving rate, n is the population growth rate, and ε_t^A is a TFP shock. Initial values for capital stock, population and TFP are given.

As we assume a positive population growth rate, all variables are defined in per capital terms.

$$y_t = A_t k_t^{\alpha} \tag{7}$$

$$k_{t+1} = \frac{(1-\delta)k_t + sy_t}{(1+n)} \tag{8}$$

where $y_t = Y_t/L_t$, $k_t = K_t/L_t$

Denoting $\Delta k_t = k_{t+1} - k_t$, and operating in the above equation, we obtain that:

$$\Delta k_t = \frac{s_t A_t k_t^{\alpha} - (n+\delta)k_t}{(1+n)} \tag{9}$$

and where per capita capital growth rate is defined as:

$$g_k = \frac{s_t A_t k_t^{\alpha} - (n+\delta)k_t}{(1+n)k_t} \tag{10}$$

Steady state The steady state is given by that situation in which the dynamic equation for the stock of capital per capita is zero, that is, the stock of capital per capita (and therefore the rest of the variables) is kept constant period by period. In this case, the growth rate of the model variables is all equal to zero. By equaling the change in per capita capital stock to zero and clearing, we obtain that the equilibrium condition is given by:

$$s\overline{y} = (n+\delta)\overline{k} \tag{11}$$

where sy is the saving or gross investment per worker in steady state and where $(n+\delta)$ is the effective depreciation rate of the capital stock per unit of capital per worker. It follows that the steady state would be given by:

$$sA_t\overline{k}_t^{\alpha} = (\delta+n)\overline{k}_t \tag{12}$$

Solving, the steady state is given by,

$$\overline{k} = \left(\frac{n+\delta}{sA}\right)^{\frac{1}{\alpha-1}} \tag{13}$$

$$\overline{y} = A\overline{k}^{\alpha} = A\left(\frac{n+\delta}{sA}\right)^{\frac{\alpha}{\alpha-1}} \tag{14}$$

$$\overline{c} = (1-s)\overline{y} = (1-s)A\left(\frac{n+\delta}{sA}\right)^{\frac{\alpha}{\alpha-1}} \tag{15}$$

$$\overline{i} = sA\left(\frac{n+\delta}{sA}\right)^{\frac{\alpha}{\alpha-1}} \tag{16}$$

2 Taking the model to Excel

The model is solved in the spreadsheet named "Solow.xlsx". The calibrated values of the parameters appear in cells "B4", "B5" and "B6". In column "C" appear those same parameters, in order to analyze the changes in them. In cells "B9" and "B10", we have introduced the value of the exogenous variables at the initial moment. In order to perform different types of analysis based on the value of the parameters, we have introduced a new column, "C", where we can change its value and automatically calculate its effects on the variables of the economy. We have named the values in column "B" as the initial situation, with a subscript 0, while we call the values in column "C" the final situation, with a subscript 1.

Next, cells "B13" through "B16" show the steady state for the stock of capital per capita, production per capita, consumption per capita and investment per capita, with the initial values of the parameters and exogenous variables. The equivalent cells in column "C" show the corresponding steady-state values calculated with the final values for the parameters and exogenous variables.

The model information appears in columns "E" to "K". Column "E" is the time index. In the "F-K" columns, we calculate the value of the relevant variables: capital stock per capita, production level per capita, per capita savings, per capita consumption, variation in the stock of capital per capita and growth rate of capital per capita. Cell "F3" is the initial steady-state value, calculated above. On the other hand, in cell "F4" we find the following expression:

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=(H3+(1-Delta1)*F3)/(1+n_1)
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This same expression appears in the remaining cells of column "F". Alternatively, we can simplify this expression and introduce the following:

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=F3+J3
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where cell "J3" calculates the variation of the stock of capital per capita in the period. Column "G" is the level of production per capita. If we place the cursor in cell "G3" the expression that appears is:

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=PTF_0*F3^Alpha_0
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which is the expression corresponding to the capital intensive production function obtained previously. In cell "G4" the expression is:

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=PTF_1*F4^Alpha_1
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to allow the possibility of performing analysis on changes in any of the parameters of the model. Column "H" contains the saving of the economy, which is simply obtained by multiplying the savings rate by the level of production. Column "I" is per capita consumption, which is obtained as the difference between the two previous columns, that is, the difference between what is produced and what is saved. Column "J" shows the variation in the stock of capital per capita. If we place the cursor in cell "J3", the expression that appears is,

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=(H3-(n_0+Delta_0)*F3)/(1+n_0)
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This same expression appears in the following rows of this column, but referred to the values of the parameters and exogenous variables in the final situation. Finally, column "K" contains the expression for the growth rate of per capita capital stock.

3 Exercises

1. Suppose that an earthquake reduces the stock of capital by 10% (without causing population losses). Using the "Solow.xlsx" spreadsheet, study the effects of this disturbance (Hint: change the value of cell G4 and enter in this cell the expression "=0.9*kss0").

2. Analyze the effects of an increase in the technological parameter that determines the elasticity of the production level with respect to the stock of capital (for example, suppose that the new value of α is of 0.40). What happens for a value of $\alpha = 1$ (in this case the returns on capital would be constant)? How the dynamics of the economy change in the face of this assumption? Why is this result obtained?

3. Although the physical depreciation rate of capital is assumed to be a parameter that remains constant over time, in practice its value depends on the type of capital asset in which it is invested. However, the type of capital assets changes over time. In fact, investment in capital assets related to new technologies usually have high depreciation rates. This means that the rate of physical depreciation of capital is altered over time by the different composition of capital, increasingly. Assume that the physical depreciation rate of capital increases to 8 percent per year (change the value of cell "C5" to 0.08). What consequences this increase in the physical depreciation rate of capital has on the economy? How this change affects the rate of growth of the economy?

4. Suppose that the initial savings rate, with the parameters given in the spreadsheet "Solow.xlsx", is 20%. Compare this situation with a savings rate of 30%. In what situation output per capita is higher? And the per capita consumption. Check now what happens with per capita consumption if the savings rate increases to 40%. What causes this behavior? In view of these results, what would be the savings rate that generates the highest level of welfare, that is, the one that generates the highest level of per capita consumption (this is what is called the gold-saving rate)? What relationship does the gold-saving rate have with the technological parameter that determines the elasticity of the production level with respect to the capital stock?

5. Study the effects of an increase in total factor productivity (TFP). For example, suppose the TFP increases to 1.05. What effects does this change have on the economy? What proportion of the level of production in the new steady state is due to the increase in the TFP and what part to the process of generated capital accumulation?