The Dornbusch exchange rate overshooting model

1 The model

One of the most popular dynamic non-micro-founded models with rational expectations in macroeconomics and, in particular, in international economics, is the model of overreaction or overshooting of the exchange rate developed by Rudiger Dornbusch and published in 1976. This model is a dynamic extension of the Mundell-Fleming model, or IS-LM model for an open economy, which attempts to explain the significant fluctuations observed in nominal changes. It is a model for a small open economy with perfect international capital mobility. The model considers the existence of two markets: the market for goods and services and the money market, as in the IS-LM model for a closed economy. In an open economy, we have two equilibrium relations at the international level, which will relate the domestic economy to the rest of the world. These international-equilibrium relations connect both domestic markets with their equivalents abroad. The equilibrium relationship between the national and foreign money markets is determined through the Uncovered Interest Parity (UIP), which establishes that the expectation of depreciation of the (log of) nominal exchange rate is equal to the difference between the domestic and foreign nominal interest rates. The equilibrium relationship between domestic and foreign goods and services markets is determined by the Purchasing Power Parity (PPP), which establishes that the (log of) nominal exchange rate is equal to the difference between the (log of) domestic and foreign prices. Given the existence of rigidities in the adjustment of prices, the Purchasing Power Parity is only fulfilled in the long-run.

The rigidity in the adjustment of prices will explain the phenomenon of overshooting of the exchange rate. For instance, in the face of an increase in money supply, the exchange rate increases instantaneously to a value higher than its new steady-state value, and then progressively decreases as prices adjust to the new steady state. In effect, the increase in the amount of money immediately causes a decrease in the national interest rate, and given the UIP, there is an appreciation of the nominal exchange rate. However, in the long-run, the increase in money supply will lead to a higher price level, so the exchange rate will also be higher. In order for the decrease in the exchange rate to be compatible with a higher value in the long term, there must be an initial overshooting in the exchange rate, reaching a value higher than its new steady-state value, to subsequently decrease to reach its new steady state.

The structure of the economy is given by the following four equations:

$$m_t - p_t = \psi y_t - \theta i_t \tag{1}$$

$$y_t^d = \beta_0 + \beta_1(s_t - p_t + p_t^*) - \beta_2 i_t \tag{2}$$

$$\Delta p_t = \mu(y_t^d - y_t^n) \tag{3}$$

$$\Delta s_t^e = \upsilon(i_t - i_t^*) \tag{4}$$

where all variables are defined in logarithms, except the nominal interest rate, and where m denotes the quantity of money, p is the price, y is output, i is the nominal interest rate, y^d is the aggregate demand, and y^n is potential output. A start over the variables defines foreign variables (assumed to be exogenous given the assumption of a small open economy). The symbol Δ defines the variation of the corresponding variable between two periods. s^e represents the expected value of the exchange rate, that is, expectations about the future value of the exchange rate. Since we solve the model in a context of perfect foresight and under the assumption that expectations are rational, then we have that $s^e = s$. We define the nominal exchange rate as the local currency cost of one unit of foreign currency.

The first equation (expression 1) is the demand for money, where the demand for real balances depends positively on the level of production and negatively on the nominal interest rate. This equilibrium condition for the money market does not change with respect to what we would have in a closed economy. The second equation (expression 2) is the aggregate demand of an open economy. When we define the aggregate demand of an open economy, we need to take into account not only the internal component but also the external component (net exports). We assume that the aggregate demand of an open economy depends positively on the real exchange rate, defined as the deviations from Purchasing Power Parity (the real exchange rate), that reflects the level of external competitiveness by prices of the economy. On the other hand, we assume that aggregate demand depends negatively on the nominal interest rate, instead of depending negatively on the real interest rate that would be the correct statement. We have made this simplification because the results will not be altered if instead of the real interest rate we consider the nominal interest rate, although we do not know this a priori. The other two equations are dynamic equations of price adjustment as a function of the difference between aggregate demand and potential output (expression 3), and the nominal exchange rate adjustment equation (4), represented by the Uncovered Interest Parity (the equilibrium relationship between domestic and foreign money markets).

The above system of equations contains 5 endogenous variables: domestic prices, domestic output, domestic nominal interest rate, aggregate demand and nominal exchange rate. On the other hand, we have 5 exogenous variables: money, public spending, foreign price, potential output and foreign nominal interest rate. Although the number of exogenous variables included in the model can be any (their value is predetermined), the number of endogenous variables must coincide with the number of available equations. To equal the number of endogenous variables with the number of equations, we assume that domestic output is always equal to the domestic potential ($y_t = y_t^n$). Finally, inflation is defined as

$$\Delta p_t = p_{t+1} - p_t \tag{5}$$

and the rate of depreciation of the exchange rate as

$$\Delta s_t = s_{t+1} - s_t \tag{6}$$

Solution To solve this model analytically, as a previous step to its numerical resolution, we will proceed by reducing the model to a system of two difference equations, in terms of price and output. To obtain these two equations for inflation and output growth, we must first solve for the rest of the endogenous variables, that is, nominal interest rate and aggregate demand. To obtain the nominal interest rate, we solve equation (1):

$$i = -\frac{1}{\theta}\left(m_t - p_t - \psi y_t\right) \tag{7}$$

to obtain the value of the nominal interest rate as a function of the endogenous variables to calculate (prices and output) and the exogenous variables (quantity of money). Next, we will solve for the aggregate demand. As we can see in equation (2), the nominal interest rate appears as a variable driving aggregate demand. Replacing equation (7) in (2), so that we obtain:

$$y_t^d = \beta_0 + \beta_1(s_t - p_t + p_t^*) + \frac{\beta_2}{\theta}(m_t - p_t - \psi y_t^n) \tag{8}$$

and again, by substituting this equation in the expression (3), we obtain the dynamic equation for the domestic price:

$$\Delta p_t = \mu\beta_0 + \mu\beta_1 s_t + \mu\beta_1 p_t^* - \mu(\beta_1 + \frac{\beta_2}{\theta})p_t + \frac{\mu\beta_2}{\theta}m_t - \mu(\frac{\psi\beta_2}{\theta} + 1)y_t^n \tag{9}$$

On the other hand, by substituting the equation (6) in the expression (4), we obtain the dynamic equation for the nominal exchange rate:

$$\Delta s_t = -\frac{1}{\theta}(m_t - p_t - \psi y_t^n) - i_t^* \tag{10}$$

The model can be represented by the following system of linear difference equations:

$$\begin{aligned}\begin{bmatrix}\Delta p_t \\ \Delta s_t\end{bmatrix} &= \underbrace{\begin{bmatrix}-\mu(\beta_1 + \frac{\beta_2}{\theta}) & \mu\beta_1 \\ \frac{1}{\theta} & 0\end{bmatrix}}_{A}\begin{bmatrix}p_t \\ s_t\end{bmatrix} + \\ &\underbrace{\begin{bmatrix}\mu & \frac{\mu\beta_2}{\theta} & -\mu(\frac{\psi\beta_2}{\theta}+1) & 0 & \mu\beta_1 \\ 0 & -\frac{1}{\theta} & \frac{\psi}{\theta} & -1 & 0\end{bmatrix}}_{B}\begin{bmatrix}\beta_0 \\ m_t \\ y_t^n \\ i_t^* \\ p_t^*\end{bmatrix}\end{aligned} \tag{11}$$

where A is the matrix of parameters for the endogenous variables and B is the matrix of parameters of the exogenous variables.

Steady state Steady state of the economy is defined as the value for the endogenous variables for which they remain constant over time, that is, inflation and output growth are zero. Steady state is calculated by simply equating the dynamic equations of the system to zero, indicating that the variation over time of the endogenous variables is zero, so the variables are constant period to period. Using the model in matrix notation, the vector variables in steady state would be defined as:

$$\begin{bmatrix} \overline{p} \\ \overline{y} \end{bmatrix} = -A^{-1}B \begin{bmatrix} \beta_0 \\ m_t \\ y_t^n \\ i_t^* \\ p_t^* \end{bmatrix} \tag{12}$$

where steady state values are denoted by an upper bar on the variable, being the inverse of matrix A:

$$A^{-1} = \begin{bmatrix} 0 & \theta \\ \frac{1}{\mu\beta_1} & \frac{\beta_1\theta+\beta_2}{\beta_1} \end{bmatrix} \tag{13}$$

Operating,

$$-A^{-1}B = \begin{bmatrix} 0 & 1 & -\psi & \theta & 0 \\ -\frac{1}{\beta_1} & 1 & \frac{(1-\beta_1\psi)}{\beta_1} & \frac{\beta_1\theta+\beta_2}{\beta_1} & -1 \end{bmatrix} \tag{14}$$

Steady state of the economy is defined by:

$$\overline{p} = m - \psi y^n + \theta i^* \tag{15}$$

$$\overline{s} = m - \frac{\beta_0}{\beta_1} + \left[\frac{1-\psi\beta_1}{\beta_1}\right] y^n + \frac{\theta\beta_1 + \beta_2}{\beta_1} i^* - p^* \tag{16}$$

Eigenvalues The stability of the system is determined by calculating $|A - \lambda I| = 0$, where A is the matrix of parameters defined above, λ represents the vector of eigenvalues and I is the identity matrix, being:

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{17}$$

In our case, we would have:

$$Det \begin{bmatrix} -\mu(\beta_1 + \frac{\beta 2}{\theta}) - \lambda & \mu\beta_1 \\ \frac{1}{\theta} & 0 - \lambda \end{bmatrix} = 0 \tag{18}$$

from which we obtain:

$$\lambda^2 + \lambda \left[\beta_1\mu + \frac{\beta_2\mu}{\theta}\right] - \frac{\beta_1\mu}{\theta} = 0 \tag{19}$$

The solution of the system is a saddle point, with one eigenvalue negative and the other positive. Eigenvalues are,

$$\lambda_1, \lambda_2 = \frac{-(\beta_1\mu + \frac{\beta_2\mu}{\theta}) \pm \sqrt{\left[(\beta_1\mu + \frac{\beta_2\mu}{\theta})\right]^2 + \frac{4\beta_1\mu}{\theta}}}{2} \tag{20}$$

Jumper forward-looking variable adjustment A key element for the numerical solution of this model, given that the solution is a saddle point, is the calculation of the so-called stable saddle path, which is a stable trajectory that converges directly to the steady state. In this case, the dynamics of the economy in the adjustment from an initial steady state to another steady state when a shock occurs, is represented by an instantaneous readjustment to the new stable saddle path by the "jump" variable. Once this stable path is reached, the economy moves along it until it reaches the new steady state. This stable path can be calculated as a trajectory that is associated with the eigenvalue whose module plus one is less than the unit. Thus, in mathematical terms, the trajectory that defines the stable saddle path can be calculated through the following dynamic system:

$$\begin{bmatrix} \Delta p_t \\ \Delta s_t \end{bmatrix} = \lambda_1 \begin{bmatrix} p_t - \overline{p} \\ s_t - \overline{s} \end{bmatrix} \tag{21}$$

where λ_1 is the stable eigenvalue.

In the particular case of this model, the forward-looking variable is the nominal exchange rate. Therefore, when a disturbance occurs, an instantaneous readjustment occurs in the nominal exchange rate, which is caused by a readjustment in its expectations about its future value. This readjustment in expectations is instantaneously transferred to its current value. If agents in the foreign exchange market expect that in the future the exchange rate will be higher, they will readjust their expectations about the future exchange rate, which will result in an increase in the exchange rate. This readjustment in expectations cannot be calculated directly from the equations of the model since we have applied the perfect foresight assumption in such a way that the expected change in the exchange rate is equal to the current depreciation. Therefore, to have a numerical solution of the model it is necessary to calculate the new value of the nominal exchange rate consistent with the new stable saddle path when a shock hits the economy. This jump in the nominal exchange rate to the new stable saddle path is what it is known as the overshooting phenomenon. To calculate the jump in the nominal exchange rate following a shock, we depart from the dynamic equation of the exchange rate:

$$\Delta s_t = -\frac{1}{\theta}(m_t - p_t - \psi y_t^n) - i_t^* \tag{22}$$

In parallel, we can define the stable trajectory that will follow the nominal exchange rate associated with the stable root (λ_1), given by:

$$\Delta s_t = \lambda_1(s_t - \overline{s}_t) \tag{23}$$

As we can see, both equations result in the variation with respect to the time of the nominal exchange rate, so we can equalize both equations at the time in which the disturbance occurs ($t = 1$):

$$\lambda_1(s_1 - \overline{s}_1) = -\frac{1}{\theta}(m_1 - p_1 - \psi y_1^n) - i_1^*$$

Solving for the value of the exchange rate results in the following expression, corresponding to the value of the jumping variable (the forward-looking variable, i.e., nominal exchange rate) in the instant when a shock hits the economy,

$$s_t = \frac{-(m_1 - p_1 - \psi y_1^n)}{\theta\lambda_1} - \frac{i_1^*}{\lambda_1} + \overline{s}_1 \tag{24}$$

where λ_1 is the stable eigenvalue, in order the economy be at the stable saddle path to the new steady state.

2 Taking the model to Excel

The model is solved in the Excel file "Dornbusch.xlsx". Columns "F", "G", "H" and "I" show the value of each one of the endogenous variables (prices, nominal exchange rate, aggregate demand and nominal interest rate) in every moment of time. The calibrated values of the parameters appear in cells "B10" to "B14". The values of the exogenous variables appear in cells "B17" to "B21", which we have named "`m_0`", "`iext_0`", "`Beta0_0`", "`pext_0`" and "`ypot_0`", respectively. The initial steady-state values appear in cells "B24" and "B25". Cells "C24" and "C25" show the new steady state in the case where a disturbance occurs (change in the exogenous variables). The value of the eigenvalues is given in rows 28 and 29. In cells "B28" and B29 "the real part is shown, while the imaginary part is shown in cells" C28 "and" C29". Finally, the root modules are shown in cells "B32" and "B33".

In cell "F3" we find the following expression:

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=pbar_0
```

which is simply the expression corresponding to the initial steady-state value of the domestic price level given in cell "B24". The remaining rows in this column simply contain the value of the domestic price level in the previous moment plus the change produced in that price level. Thus, cell "F4" contains the expression "`=F3+J3`", where "F3" refers to the domestic price level of the previous period and "J3" to the change in the price level. This expression is copied into the remaining rows of that column.

On the other hand, if we place the cursor in cell "G3" it contains the expression:

```
=sbar_0
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that is, the initial steady-state value of the nominal exchange rate given by cell "B25". Next, a somewhat special cell appears, the "G4", in which we have to introduce the value of the nominal exchange rate corresponding to the jump to the stable saddle path. In cell "G5", the expression "`=G4+K4`" appears, in which we define the nominal exchange rate for each period as the previous one plus the change experienced in it. Column "H" contains the aggregate-demand values. If we place ourselves in cell "H3", we see that this expression appears:

```
=Beta0_0+Beta1*(G3-F3+pext_0)+(Beta2/Theta)*(m_0-F3-Psi*ypot_0)
```

which corresponds to the aggregate demand equation of the model. This same expression appears in the following cells in this column. Column "I" contains the domestic nominal interest rate values. Thus, cell "I3" contains the following expression:

```
=-1/Theta*(m_0-F3-Psi*ypot_0)
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which is the equation resulting from solving for the interest rate from the money-demand equation. If we place ourselves in cell "I4", the expression that appears is:

```
=-1/Theta*(m_1-F4-Psi*ypot_1)
```

which refers to the new amount of money from time 1. This expression is the same one that appears in the following rows of this column.

Finally, columns "J" and "K" show the variations in domestic prices and the nominal exchange rate, that is, they define the value of inflation and the depreciation of the nominal exchange rate in each period. In this case, we must introduce the corresponding equations that determine the behavior of both variables. If we place ourselves in cell "J3" we see that it contains the expression:

```
=Mi*(H3-ypot_0)
```

while cell "J4" contains the expression:

```
=Mi*(H4-ypot_1)
```

being this same expression the one that appears in the following cells, since it is possible that we want to analyze the effects of an alteration in the level of potential output of the domestic economy. On the other hand, if we place ourselves in cell "K3", we see that it contains the expression:

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=I3-iext_0
```

which corresponds to the dynamic equation for the nominal exchange rate (exchange rate depreciation). As we can see in the spreadsheet, we can enter the initial expression given by the model, since we will also calculate the corresponding value of the aggregate demand at each moment of time. If all calculations are correct, columns "J" and "K", where the change of each variable appears, must be zeros in the initial steady state.

Columns "L" to "O" show the variables in levels. This is done by applying the antilogarithm to the variables in logs, except to the nominal interest rate. For example, in cell "L3" we find the expression "`=exp(F3)`", which transforms the log of the price into the price level. Finally, columns "P" to "S" show the deviation of the variables in levels, in percentage points, with respect to the initial steady state (basic points for the nominal interest rate)

Finally, notice that the value of the exchange rate at the time in which the disturbance occurs (at period 1), representing the jump in this variable to the stable saddle path, is introduced in cell "G4".

3 Excercises

1. Using the "Dornbusch.xlsx" spreadsheet, The Model tab, study the effects of an increase in public spending, represented by β_0. Specifically, suppose that public spending increases by 0.1 units. Does the phenomenon of overshooting of the exchange rate occur in this case? (Hint: change the value of cell "C19" from 5 to 5.1).

2. Suppose that a technological change occurs and increases the level of potential output. Use the spreadsheet to study the effects of this disturbance on the economy. What is the transition dynamic to the new steady state? (Hint: for example, change the value of cell "C21" from 10 to 11).

3. Study the effects of an increase in the foreign interest rate from a value of 3% to a value of 4%. (Hint: change the value of cell "C18" from 0.03 to 0.04).